Find the curve's unit tangent vector $\vec{r}(t)$

unit tangent vector $T = \frac{dr}{ds} = \frac{dr/dt}{ds/dt} = \frac{v}{|v|}$

Length $L = \int_a^b |\vec{v}|\, dt$

$|\vec{v}| = \sqrt{V_1^2 + V_2^2 + V_3^2}$

1. find v $i + j + k$ (derivative with respect to t)
2. Find $|\vec{v}|$ Square each term
3. Simplify the radical $\cos^2\theta + \sin^2\theta = 1$
4. find T $i + j + k$
5. Find length L $3\sqrt{2}t \Big|_0^{\pi}$ $3\sqrt{2}\pi - 3\sqrt{2}(0)$

product rule $(f \cdot g)' = f' \cdot g + f \cdot g'$ $3t \cos t$

Find the arc length parameter along the given curve from P where $t = 0$.

1. find $v(t)$ (derivative) $i + j + k$
2. find length of $|\vec{v}(t)|$ velocity vector
3. Find arc length $s(t)$ $\int_0^t |\vec{v}(t)|\, dt$

0 to t then 0 to π then diff. between $t = 0$ and $t = \pi$

Calculate the length of one turn of the helix with the following parameterizations

1. Find $\vec{v}$, derivative $r(t)$. $i + j + k$
2. find length of velocity vector $|\vec{v}(t)|$
3. Find arc length parameter $L = \int_{t_0}^{t} |\vec{v}(t)|\, dt$

may need chain rule $f'(g(x)) = f'(g(x))\, g'(x)$

Find T, N, and K for the plane curve $\vec{r}(t)$

1. Find velocity vector $\vec{v} = r'(t)$ $i + j + k$
2. Find length $|\vec{v}|$ $1 + \tan^2 t = \sec^2 t$ $\cos t = \frac{1}{\sec t}$
3. Find unit tangent vector $T = \frac{v}{|v|}$ may need chain rule
4. Find dT/dt (derivative of $T(t)$)
5. Find $|dT/dt|$
6. Principal normal vector $N = \frac{dT/dt}{|dT/dt|}$ $i\, j\, k$
7. Find curvature $\kappa = \frac{1}{|v|} \cdot \left|\frac{dT}{dt}\right|$

Find the total curvature of the helix

↑ 1. steps 1 to 5
2. Perform integration $K = \int_{t_0}^{t} \left|\frac{dT}{dt}\right| dt$

Find the total curvature of the parabola

1. Parametize $y = 2x^2 \rightarrow r(t) = i + 2t^2 j$

↑ 2. steps 1 to 5 $\frac{1}{1+x^2} = \tan^{-1} x$

Find an equation for the circle of curvature

1. find $\vec{v}$ and $|\vec{v}|$ $k = \frac{|x'y'' - y'x''|}{\left[1 + (y'(x))^2\right]^{3/2}}$

maximum curvature $k(x) = \frac{|f''(x)|}{\left[1 + (f'(x))^2\right]^{3/2}}$

Function	Domain	Range
$z = \sqrt{y - x^2}$	$y \geq x^2$	$[0, \infty)$
$z = \frac{1}{xy}$	$xy \neq 0$	$(-\infty, 0) \cup (0, \infty)$
$z = \sin xy$	Entire plane	$[-1, 1]$

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Function	Domain	Range
$w = \sqrt{x^2 + y^2 + z^2}$	Entire space	$[0, \infty)$
$w = \frac{1}{x^2 + y^2 + z^2}$	$(x, y, z) \neq (0, 0, 0)$	$(0, \infty)$
$w = xy \ln z$	Half-space $z > 0$	$(-\infty, \infty)$

Find and sketch the level curves $f(x,y) = c$.

1. substitute c in $f(x,y) = c$.
2. Solve for y

Find lim 1. if $f(x)$ is defined, plug in values. else: factor and simplify, then plug values.

two path test 1. find lim $x \rightarrow 0^+$ at $y = x$ then $x \rightarrow 0^-$ at $y = x$.

for $x \rightarrow 0^+$ $|x|$ for $x \rightarrow 0^-$ $-x$.

1. if $f(x,y) = \frac{2x^2 y}{x^4 + y^2}$ undefined at $(0,0)$ use $y = kx^2$
2. simplify then find value at $k = 1$, $k = 0$ if lims $\neq$ then lim does not exist.

Find $\partial f/\partial x$ and $\partial f/\partial y$.

for $\partial f/\partial x$ treat y as a constant $4y^2 = 0$, $8xy = 8y$

for $\partial f/\partial y$ treat x as a constant $5x^9 = 0$, $8xy = 8x$

may need chain rule $f'(g(x)) = f'(g(x))\, g'(x)$

product rule $(f \cdot g)' = f' \cdot g + f \cdot g'$

$\frac{\partial}{\partial x}(a^x) = a^x \ln a$

second order partial derivatives

$\frac{\partial^2 g}{\partial x^2}$ Find $\frac{\partial g}{\partial x}'$ by taking derivative of $g(x,y)$ treating y as a constant. then $\frac{\partial g}{\partial x}''$

$\frac{\partial^2 g}{\partial y \partial x} \rightarrow \frac{\partial}{\partial y}\frac{\partial g}{\partial x}$ take derivative of dg/dx treating x as a constant.

$\frac{\partial^2 g}{\partial y^2}$ Find $\frac{\partial g}{\partial y}'$ by taking derivative of $g(x,y)$ treating x as a constant. then $\frac{\partial g}{\partial y}''$

$\frac{\partial^2 g}{\partial x \partial y} \rightarrow \frac{\partial}{\partial x}\frac{\partial g}{\partial y}$ take derivative of dg/dy treating y as a constant.

use the limit definition of partial derivatives to compute the partial derivative of $f(x,y) = 7 - 2x + 5y - 4x^2 y$ at $(3, 2)$.

Find $\frac{\partial f}{\partial x}$ at $(3,2)$ $f_x(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0^{3} + h, y_0^{2}) - f(x_0, y_0)}{h}$

1. $f(3 + h, 2) \rightarrow$ plug $x = (3+h)$ $y = 2$ in $f(x,y)$

$(3+h)^2 = a^2 + 2ab + b^2$ ANS $-61 - 50h - 8h^2$

2. $f(3,2) \rightarrow$ plug $x = 3$, $y = 2$ in $f(x,y)$ ANS -61.

3. apply $f_x(x_0,y_0)$ formula ANS -50.

Find $\frac{\partial f}{\partial y}$ at $(3,2)$ $f_y(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0^{3}, y_0^{2} + h) - f(x_0, y_0)}{h}$

1. $f(3, 2+h) \rightarrow$ plug $x = 3$ $y = (2+h)$ in $f(x,y)$ ANS $-61 - 31h$

2. apply $f_y(x_0,y_0)$ formula ANS -31.

Find the slope of the tangent line to $f(x,y)$ at P. lying in plane $x = 1$ $y = 1$.

1. take $\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}$ treat x as constant
2. evaluate result at $x = 1, y = 1$ $4y^3 \big|_{(1,1)}$
3. take $\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}$ treat y as constant
4. evaluate result at $x = 1, y = 1$ $3x^2 \big|_{(1,1)}$

For the functions $w = 9x^2 + 3y^2$, $x = \cos t$, and $y = \sin t$. express $\frac{dw}{dt}$ as a function of t, by using the chain rule and by expressing w in terms of t and differentiating directly with respect to t. Then, evaluate $\frac{dw}{dt}$ at $t = \frac{\pi}{2}$.

Express $\frac{dw}{dt}$ as a function of t.

Partial differentiate w with respect to x treat y as a constant $\frac{\partial w}{\partial x} = \frac{\partial}{\partial x}$

Partial differentiate w with respect to y treat x as a constant $\frac{\partial w}{\partial y} = \frac{\partial}{\partial y}$

differentiate $x = \cos t$ with respect to t $\frac{dx}{dt} = \frac{d}{dt}(\cos t)$

differentiate $y = \sin t$ with respect to t $\frac{dy}{dt} = \frac{d}{dt}(\sin t)$

write $\frac{dw}{dt}$ using the chain rule $\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}$

plug $x = \cos t$, $y = \sin t$ after simplifying

Evaluate $\frac{dw}{dt}$ at $t = \frac{\pi}{2}$. $\frac{dw}{dt}\Big|_{t = \frac{\pi}{2}}$

consider the functions $z = -3e^x \ln y$, $x = \ln(u \cos v)$, and $y = u \sin v$.

a) express $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ as functions of both u and v by using the chain rule and by expressing z directly in terms of u and v before differentiating.

b) evaluate $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ at $(u,v) = \left(3, \frac{\pi}{3}\right)$

$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$ Partial differentiate z with respect to x $\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}$

Partial differentiate x with respect to u $\frac{\partial x}{\partial u} = \frac{\partial}{\partial u}$ $\frac{\partial}{\partial x}(\ln x) = \frac{1}{x}$

may need chain rule afuera . adentro $f'(g(x)) = f'(g(x))\, g'(x)$

Partial differentiate z with respect to y $\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}$

Partial differentiate y with respect to u $\frac{\partial y}{\partial u} = \frac{\partial}{\partial u}$

cont. apply formula, insert values of x and y into equation, Factor. $e^{\ln x} = \ln$

b) evaluate $\partial z/\partial u$ at $(u,v) = (3, \pi/3)$ $\frac{\partial z}{\partial u}\Big|_{(3,\pi/3)}$

repeat all steps but dz/dx and dz/dy since you already have those values. $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$

evaluate $\frac{\partial z}{\partial v}$ at $(u,v) = \left(3, \frac{\pi}{3}\right)$ $\frac{\partial z}{\partial v}\Big|_{(3,\pi/3)}$

assuming $4x^3 - y^2 - 3xy = 0$ defines y as a differentiable function of x, use the theorem $\frac{dy}{dx} = -\frac{F_x}{F_y}$ to find $\frac{dy}{dx}$ at the point $(1,1)$.

differentiate $F(x,y)$ with respect to x. $F_x = \frac{\partial}{\partial x}$ treat y as a constant

differentiate $F(x,y)$ with respect to y. $F_y = \frac{\partial}{\partial y}$ treat x as a constant

write $\frac{dy}{dx}$ using the theorem $\frac{dy}{dx} = -\frac{F_x}{F_y}$ $\frac{dy}{dx}\Big|_{(1,1)}$

substitute $x = 1$ and $y = 1$ and evaluate at the given point.

Find $\frac{dw}{dr}$ when $r = 2$ and $s = -2$ if $w = (x + y + z)^2$

$x = r - s$, $y = \cos(r + s)$, $z = \sin(r + s)$

$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r}$ Find all derivatives then plug into formula.

solve for $x = r - s$, $y = \cos(r+s)$, $z = \sin(r+s)$ by plugging values of r and s.

go back to formula and plug x,y,z values. $\frac{\partial w}{\partial r}\Big|_{\substack{r=2 \\ s=-2}}$

assume that $w = f(s^3 + t^2)$ and $f'(x) = e^x$. Find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$

differentiate w with respect to t $\frac{\partial w}{\partial t}$ may need chain rule

substitute $x = (s^3 + t^2)$ in $f'(x) = e^x$

substitute $f'(s^3 + t^2) = e^{s^3 + t^2}$ in $2t f'(s^3 + t^2)$

assume that $z = f(w)$ $w = g(x,y)$

$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r}$ take derivative of $\frac{\partial x}{\partial r}$ take derivative of $\frac{\partial y}{\partial r}$

z/r g_x g_y plug g_x g_y.

$\frac{\partial z}{\partial r} = f'(w) \cdot w$ then plug $r = s$, $s = 0$

$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s}$ take all steps.

Find the gradient of the function $f(x,y)$ at P.

$\vec{\nabla} f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$ $\perp$ to surface $f(x,y)$

take partials, then plug point. $\Delta f \big|_{x,y}$ $i + j + k$

derivative of function P_0 in the direction of A.

take partials, then plug point. $\Delta f \big|_{x,y}$ gradient

you have vector A. Find $|A|$ $i + j + k$

then use formula $u = \frac{A}{|A|}$, u_x "i" and u_y "j"

$D_A f = \frac{\partial f}{\partial x} u_x + \frac{\partial f}{\partial y} u_y$

$\frac{d}{dx}\left[\frac{f(x)}{f(y)}\right] = \frac{g(x) f'(x) - f(x) g'(x)}{[g(x)]^2}$

Find the direction in which func. increases and decreases at P.

take partials, then plug point. $\Delta f \big|_{x,y}$ gradient

you have vector Δf. Find $|\Delta f|$ $i + j + k$

then formula $u = \frac{\Delta f(x,y)}{|\Delta f|(x,y)|} = i + j + k$. $+ = u$ $- = -u$

tangent line equation $f_x(x_0)(x - x_0) + f_y(y_0)(y - y_0) = 0$ from gradient vector

find parametric eq. for line perpendicular to the graph of eq. $5x^2 + 5y^2 + 5z^2 = 105$ at the point $(4, -2, -1)$.

1. find gradient $\nabla f = \frac{\partial F}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial F}{\partial z} k$
2. evaluate at P. $\nabla F\big|_{(4,-2,-1)}$ $i + j + k$
3. find parametric eq. plug $(x_0, y_0, z_0) = (4, -2, -1)$ into $x = x_0 + 40t$ $y = y_0 - 20t$ $z = z_0 - 10t$.
4. eq. is $r(t) = (4 + 40t)i + (-2 - 20t)j + (-1 - 10t)k$

find the equation for the tangent plane and the normal line at $P(1,3,3)$ on F_x.

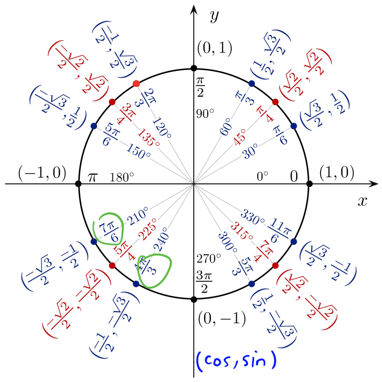
The linearization of a function f(x,y) at a point (x_0, y_0), where f is differentiable,

$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x - x_0) + f_y(x_0,y_0)(y - y_0)$

1. Find $F(x_0, y_0)$ = plug point into eq. 2. Find partial derivatives 3. Plug in point into derivative 4. plug ans. into formula. $L(x,y) = 10x + 10y - 49$

Differentiation Rules	
Constant Rule	$\frac{d}{dx}[c] = 0$
Power Rule	$\frac{d}{dx}x^n = nx^{n-1}$
Product Rule	$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
Quotient Rule	$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
Chain Rule	$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$

Derivative	Integral (Antiderivative)
$\frac{d}{dx}n = 0$	$\int 0\,dx = C$
$\frac{d}{dx}x = 1$	$\int 1\,dx = x + C$
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$
$\frac{d}{dx}e^x = e^x$ $\quad \frac{d}{dx}e^u = e^u \cdot u'$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}\ln x = \frac{1}{x}$ $\quad \frac{d}{dx}\ln u = \frac{u'}{u}$	$\int \frac{1}{x}\,dx = \ln x + C$
$\frac{d}{dx}n^x = n^x \ln x$	$\int n^x\,dx = \frac{n^x}{\ln n} + C$
$\frac{d}{dx}\sin x = \cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}\cos x = -\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$
$\frac{d}{dx}\sec x = \sec x \tan x$	$\int \tan x \sec x\,dx = \sec x + C$
$\frac{d}{dx}\csc x = -\csc x \cot x$	$\int \cot x \csc x\,dx = -\csc x + C$
$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$
$\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$	$\int -\frac{1}{\sqrt{1-x^2}}\,dx = \arccos x + C$
$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}\,dx = \arctan x + C$
$\frac{d}{dx}\operatorname{arccot} x = -\frac{1}{1+x^2}$	$\int -\frac{1}{1+x^2}\,dx = \operatorname{arccot} x + C$
$\frac{d}{dx}\operatorname{arcsec} x = \frac{1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \operatorname{arcsec} x + C$
$\frac{d}{dx}\operatorname{arccsc} x = -\frac{1}{x\sqrt{x^2-1}}$	$\int -\frac{1}{x\sqrt{x^2-1}}\,dx = \operatorname{arccsc} x + C$



Degree	Radians	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\operatorname{cosec}\theta$	$\sec\theta$	$\cot\theta$
0°	0	0	1	0	-	1	-
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$ $\frac{1}{\sqrt{3}}$	2	$\frac{2\sqrt{3}}{2}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{2}$	2	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	-	1	-	0
120°	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	-2	$-\frac{\sqrt{3}}{3}$
135°	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1
150°	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	2	$-\frac{2\sqrt{3}}{3}$	$-\sqrt{3}$
180°	π	0	-1	0	-	-1	-
270°	$\frac{3\pi}{2}$	-1	0	-	-1	-	0
360°	2π	0	1	0	-	1	-

COMMON FACTORING EXAMPLES

$x^2 - a^2 = (x+a)(x-a)$

$x^2 + 2ax + a^2 = (x+a)^2$

$x^2 - 2ax + a^2 = (x-a)^2$

$x^2 + (a+b)x + ab = (x+a)(x+b)$

$x^3 + 3ax^2 + 3a^2x + a^3 = (x+a)^3$

$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$

$x^3 - a^3 = (x-a)(x^2 + ax + a^2)$

$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$

FUNDAMENTAL IDENTITIES

$\csc\theta = \frac{1}{\sin\theta}$ $\qquad \sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$

$\cot\theta = \frac{1}{\tan\theta}$ $\qquad \sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$ $\qquad 1 + \cot^2\theta = \csc^2\theta$

$\sin(-\theta) = -\sin\theta$ $\qquad \cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{\cancel{2}}{\cancel{2}\sqrt{3}} = \frac{1}{\sqrt{3}}$$

DOUBLE ANGLE FORMULAS

$\sin 2x = 2\sin x + \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$